We assume that conditional on $\bar{\mu}$ the $d(i, j)$ are independent random variables. Let $D_{i}=\{d(i, j), 1 \leq$ $j \leq M\}$ be the array of distances between $X_{i}$ and all the $Y_{j}$. Then

$$
\begin{align*}
p(D \mid \bar{\mu}) & =\prod_{i} p\left(D_{i} \mid \bar{\mu}(i)\right),  \tag{2}\\
p\left(D_{i} \mid \bar{\mu}(i)\right) & = \begin{cases}f(d(i, j)) \prod_{k \neq j} g(d(i, k)) & \text { if } \bar{\mu}(i)=j \\
\prod_{k} g(d(i, k)) & \text { if } \bar{\mu}(i)=\tau\end{cases} \\
& = \begin{cases}L(d(i, j)) \gamma\left(D_{i}\right) & \text { if } \bar{\mu}(i)=j \\
\gamma\left(D_{i}\right) & \text { if } \bar{\mu}(i)=\tau\end{cases} \tag{3}
\end{align*}
$$

in which

$$
\begin{equation*}
L(d(i, j))=\frac{f(d(i, j))}{g(d(i, j))}, \gamma\left(D_{i}\right)=\prod_{k=1}^{M} g(d(i, k)) \tag{4}
\end{equation*}
$$



FIGURE 5 The empirical pdfs $f$ and $g$ and their Gaussian approximations for links $A \rightarrow B, B \rightarrow C$ and $C \rightarrow D$.

Relations (2)-(4) constitute the signature distance statistical model. Figure 5 displays the empirical pdfs and the Gaussian approximations of $f$ and $g$ for the three links. The annotation in the left plot for link $A \rightarrow B$ means that $\mu_{f}$ and $\sigma_{f}$ are the mean and standard deviation for $f ; \mu_{g}$ and $\sigma_{g}$ are the mean and standard deviation for $g ; n_{f}=91$ and $n_{g}=24,622$ are the number of samples used to estimate the statistics for $f$ and $g$, respectively. That is, there were 91 matched vehicle pairs and 24,622 unmatched pairs. (There are invariably many more unmatched pairs.) Section 7 describes how the distributions in Figure 5 are estimated.

The expected performance of the matching function (1) and others can be calculated from the model (2)-(4), see (12).

## 5. OPTIMAL CONSTRAINED MATCHING

Minimum distance matching, $\mu_{\min D}$, given in (1) is a form of unconstrained matching. (The matchings in $(6,13)$ are also unconstrained.) Unconstrained matching may violate two constraints. First, a matching may allow duplicates: two different upstream vehicles $i_{1} \neq i_{2}$ may be matched to the same downstream

